BRAIN TEASERS

SIX CLASSIC JOURNEY AND CLOCK PUZZLES, WITH ANSWERS

EASY TO HARD | 6 PUZZLES | ANSWERS INCLUDED

Donkeys, trains, a rowing boat and three clocks. Six puzzles about getting somewhere and how long it takes, starting easy and finishing hard. They come from Amusements in Mathematics by Henry Ernest Dudeney, published in 1917, in his own words.

NO. 1 · EASY

DONKEY RIDING

During a visit to the seaside Tommy and Evangeline insisted on having a donkey race over the mile course on the sands. Mr. Dobson and some of his friends whom he had met on the beach acted as judges, but, as the donkeys were familiar acquaintances and declined to part company the whole way, a dead heat was unavoidable. However, the judges, being stationed at different points on the course, which was marked off in quarter-miles, noted the following results: the first three-quarters were run in six and three-quarter minutes, the first half-mile took the same time as the second half, and the third quarter was run in exactly the same time as the last quarter. From these results Mr. Dobson amused himself in discovering just how long it took those two donkeys to run the whole mile. Can you give the answer?

In plain English: The race is one mile, marked in quarters. The first three quarters take 6¾ minutes. The first half takes as long as the second half. The third quarter takes as long as the fourth. How long does the whole mile take?

9 minutes.

Start at the end of the race, where the facts are simplest.

1. The third and fourth quarters took the same time. Call that time **one block**.
2. The second half of the mile is the third and fourth quarters together, so it took **2 blocks**.
3. The first half took as long as the second half: **2 blocks**.
4. The first three quarters are the first half plus the third quarter: 2 blocks + 1 block = **3 blocks**. That took 6¾ minutes, so one block is 6¾ ÷ 3 = **2¼ minutes**.
5. The whole mile is 2 blocks + 2 blocks = **4 blocks**, and 4 × 2¼ = **9 minutes**.

The puzzle never says how the first half splits into its two quarters, and you do not need to know.

NO. 2 · EASY

SIR EDWYN DE TUDOR

Sir Edwyn de Tudor was going to rescue his lady-love, the fair Isabella, who was held a captive by a neighbouring wicked baron. Sir Edwyn calculated that if he rode fifteen miles an hour he would arrive at the castle an hour too soon, while if he rode ten miles an hour he would get there just an hour too late. Now, it was of the first importance that he should arrive at the exact time appointed, in order that the rescue that he had planned should be a success, and the time of the tryst was five o'clock, when the captive lady would be taking her afternoon tea. The puzzle is to discover exactly how far Sir Edwyn de Tudor had to ride.

60 miles.

An hour early at one speed and an hour late at the other means the slow ride takes 2 hours longer than the fast one. So the question is really: how long is a ride that takes 2 hours longer at 10 miles an hour than at 15?

1. At 15 miles an hour, each mile takes **4 minutes**. At 10 miles an hour, each mile takes **6 minutes**.
2. So riding slowly adds **2 minutes to every mile**.
3. The slow ride is 2 hours longer, which is **120 minutes**.
4. 120 ÷ 2 = **60 miles**.

Check it. 60 miles takes 4 hours at 15 miles an hour and 6 hours at 10. If he sets off at noon, that gets him there at 4 o'clock, an hour early, or 6 o'clock, an hour late. To arrive at 5 o'clock he must ride 60 miles in 5 hours: **12 miles an hour**. Not 12½, halfway between the two speeds, which would get him there 12 minutes early.

NO. 3 · MEDIUM

CROSSING THE STREAM

During a country ramble Mr. and Mrs. Softleigh found themselves in a pretty little dilemma. They had to cross a stream in a small boat which was capable of carrying only 150 lbs. weight. But Mr. Softleigh and his wife each weighed exactly 150 lbs., and each of their sons weighed 75 lbs. And then there was the dog, who could not be induced on any terms to swim. On the principle of "ladies first," they at once sent Mrs. Softleigh over; but this was a stupid oversight, because she had to come back again with the boat, so nothing was gained by that operation. How did they all succeed in getting across?

In plain English: The boat carries 150 lbs at most. Each parent weighs 150 lbs and each son 75 lbs. The dog will not swim, but is small enough to go in the boat with one of the boys. How do all five get across?

In 11 crossings, with the two boys doing most of the rowing.

The boat can take one parent on their own, or both boys, or one boy and the dog. The trick is that whenever a parent crosses, a boy must already be waiting on the far side to bring the boat back. The brackets show who is on the far side after each crossing.

1. Both boys cross. **(Far side: both boys)**
2. One boy rows back. **(Far side: one boy)**
3. Mr. Softleigh crosses. **(Far side: one boy and Mr. Softleigh)**
4. The boy on the far side rows back. **(Far side: Mr. Softleigh)**

5. Both boys cross. **(Far side: Mr. Softleigh and both boys)**
6. One boy rows back. **(Far side: Mr. Softleigh and one boy)**
7. Mrs. Softleigh crosses. **(Far side: both parents and one boy)**
8. The boy on the far side rows back. **(Far side: both parents)**
9. Both boys cross. **(Far side: both parents and both boys)**
10. One boy rows back for the dog. **(Far side: both parents and one boy)**
11. The boy and the dog cross. **(Everyone is across)**

Each parent costs 4 crossings: boys over, one boy back, parent over, the other boy back. Two parents is 8, and 3 more bring over the boys and the dog. That makes 11.

NO. 4 · MEDIUM

THE THREE CLOCKS

On Friday, April 1, 1898, three new clocks were all set going precisely at the same time, twelve noon. At noon on the following day it was found that clock A had kept perfect time, that clock B had gained exactly one minute, and that clock C had lost exactly one minute. Now, supposing that the clocks B and C had not been regulated, but all three allowed to go on as they had begun, and that they maintained the same rates of progress without stopping, on what date and at what time of day would all three pairs of hands again point at the same moment at twelve o'clock?

In plain English: Clock B gains a minute a day and clock C loses a minute a day. All three start at noon on 1 April 1898. When will all three next show twelve o'clock at the same moment?

At noon on 22 March 1900.

First work out how many days it takes, then find the date. The date has a trap in it.

1. A clock face only shows 12 hours. So for clock B to show twelve o'clock at noon, it must be exactly **12 hours fast**. Clock C must be **12 hours slow**.
2. 12 hours is 720 minutes. At a minute a day, both take **720 days**. Clock A always shows the right time, so on day 720 all three hands point to twelve together.
3. Now count 720 days on from 1 April 1898. One year takes you to 1 April 1899: **365 days**. (Running total: 365.)
4. Another year takes you to 1 April 1900. That is also **365 days**, because **1900 was not a leap year**. (Running total: 730.)
5. That is 10 days too far. Ten days before 1 April 1900 is **22 March 1900**.

Here is the trap. A year that divides by 4 is usually a leap year, but a year ending in 00 is only a leap year if it also divides by 400. So 1800 and 1900 were not leap years, while 2000 was. Dudeney set this puzzle in 1898 to see how many people knew. Anyone who gave 1900 a 29 February ended up a day out, on 21 March.

NO. 5 · HARD

THE TWO TRAINS

I put this little question to a stationmaster, and his correct answer was so prompt that I am convinced there is no necessity to seek talented railway officials in America or elsewhere. Two trains start at the same time, one from London to Liverpool, the other from Liverpool to London. If they arrive at their destinations one hour and four hours respectively after passing one another, how much faster is one train running than the other?

In plain English: Two trains set off at the same moment from opposite ends of a line. After they pass each other, one takes 1 more hour to finish and the other takes 4 more hours. How many times faster is the quicker train?

> **One train is running twice as fast as the other.**

> Both trains ran for the same time before they passed. Call that the time before they met. Now look at what each train does afterwards.

1. After passing, the fast train covers in **1 hour** the stretch that took the slow train **the time before they met**. So the number of hours before they met is how many times faster the fast train is. (If they met after 3 hours, it would be 3 times as fast.)
2. After passing, the slow train takes **4 hours** to cover the stretch the fast train did in **the time before they met**. So 4 hours is the time before they met, multiplied by how many times faster the fast train is.
3. Put the two together: how many times faster, multiplied by itself, makes 4. That number is **2**.
4. So the fast train is twice as fast, and the trains met after **2 hours**.

> Check it with real numbers. Say the line is 180 miles, the fast train does 60 miles an hour and the slow one 30. After 2 hours the fast train has done 120 miles and the slow one 60, which makes 180, so they pass. The fast train has 60 miles left: **1 hour**. The slow train has 120 miles left: **4 hours**.

NO. 6 · HARD

THE THREE VILLAGES

I set out the other day to ride in a motor-car from Acrefield to Butterford, but by mistake I took the road going via Cheesebury, which is nearer Acrefield than Butterford, and is twelve miles to the left of the direct road I should have travelled. After arriving at Butterford I found that I had gone thirty-five miles. What are the three distances between these villages, each being a whole number of miles? I may mention that the three roads are quite straight.

In plain English: A straight road runs from Acrefield to Butterford. Cheesebury is 12 miles off to one side of it, and closer to Acrefield. Going via Cheesebury is 35 miles. Every road is straight and every distance is a whole number of miles. How far apart are the villages?

> **Acrefield to Cheesebury is 15 miles, Cheesebury to Butterford is 20 miles, and the direct road is 25 miles.**

> Draw it first. The three roads make a triangle. Now draw a line straight from Cheesebury to the direct road, meeting it at a right angle. That line is 12 miles long, and it splits the triangle into two right-angled triangles.

> In a right-angled triangle, the longest side squared equals the other two sides squared and added together. So for each road out of Cheesebury, the stretch of direct road beside it is found like this: square the road, take away 12 × 12 = 144, and find the square root.

> The two Cheesebury roads add up to 35. The one to Acrefield is the shorter, and it must be longer than 12 miles, so it is somewhere from 13 to 17. Try each.

> 1. **13 and 22.** 13 × 13 − 144 = 25, the square of 5. But 22 × 22 − 144 = 340, which is not a square. The direct road would come to about 23.4 miles. Not a whole number.
> 2. **14 and 21.** 14 × 14 − 144 = 52. Not a square, and the direct road would be about 24.4 miles.
> 3. **15 and 20.** 15 × 15 − 144 = 81, the square of 9. And 20 × 20 − 144 = 256, the square of 16. The direct road is 9 + 16 = **25 miles**. A whole number.
> 4. **16 and 19.** 16 × 16 − 144 = 112. Not a square, and the direct road would be about 25.3 miles.
> 5. **17 and 18.** 17 × 17 − 144 = 145. Not a square, and the direct road would be about 25.5 miles.

> Only one pair works. Acrefield to Cheesebury is 15, Cheesebury to Butterford is 20, and the direct road is 25. The wrong turning cost 10 miles.

All six are from Amusements in Mathematics (1917), which is in the public domain. The worked answers are our own.