BRAIN TEASERS

SIX CLASSIC NUMBER PUZZLES, WITH ANSWERS

EASY TO HARD | 6 PUZZLES | ANSWERS INCLUDED

Six puzzles for anyone who likes a sum with a twist, starting easy and finishing hard. None of them needs more than a pencil and some patience. They come from Amusements in Mathematics by Henry Ernest Dudeney, published in 1917, in his own words.

NO. 1 · EASY

THE ABBOT'S PUZZLE

"If 100 bushels of corn were distributed among 100 people in such a manner that each man received three bushels, each woman two, and each child half a bushel, how many men, women, and children were there?"

Now, there are six different correct answers, if we exclude a case where there would be no women. But let us say that there were just five times as many women as men, then what is the correct solution?

In plain English: This puzzle was old even in 1917. It goes back to Alcuin of York, a scholar who died in 804. 100 bushels of grain are shared among 100 people: 3 to each man, 2 to each woman and half to each child. There are five times as many women as men. How many of each are there?

5 men, 25 women and 70 children.

Guess a number of men, see how close it gets, and adjust.

1. **Try 1 man.** That means 5 women, and 100 − 6 = 94 children. Bushels: 3 + 10 + 47 = **60**. Too few.
2. **Try 2 men.** That means 10 women and 88 children. Bushels: 6 + 20 + 44 = **70**. Still too few, but 10 more than last time.
3. Each extra man brings 5 more women with him, which is 3 + 10 = 13 more bushels. But there are 6 fewer children, which is 3 fewer bushels. So **every extra man adds 10 bushels**.
4. We have 70 and need 100, so we need 3 more men.
5. **5 men.** That means 25 women and 70 children. Bushels: 15 + 50 + 35 = **100**. Right.

NO. 2 · MEDIUM

THE HONEST DAIRYMAN

An honest dairyman in preparing his milk for public consumption employed a can marked B, containing milk, and a can marked A, containing water. From can A he poured enough to double the contents of can B. Then he poured from can B into can A enough to double its contents. Then he finally poured from can A into can B until their contents were exactly equal. After these operations he would send the can A to London, and the puzzle is to discover what are the relative proportions of milk and water that he provides for the Londoners' breakfast-tables. Do they get equal proportions of milk and water, or two parts of milk and one of water, or what? It is an interesting question, though, curiously enough, we are not told how much milk or water he puts into the cans at the start of his operations.

Three parts water to one part milk, whatever amounts he starts with.

Dudeney does not give the amounts, so choose some. Say can B starts with 4 pints of milk and can A with 8 pints of water. The brackets show what is in each can after every pour.

1. Start. **(A: 8 water. B: 4 milk.)**
2. Pour from A until B has doubled. B needs 4 more pints, all water. **(A: 4 water. B: 4 milk and 4 water.)**
3. Pour from B until A has doubled. A needs 4 more pints. B is half milk and half water, so those 4 pints are 2 of milk and 2 of water. **(A: 2 milk and 6 water. B: 2 milk and 2 water.)**
4. Pour from A until the cans are equal. There are 12 pints in all, so 6 each, and 2 pints go across. Can A has less in it now, but the mix is the same. **(A: 1½ milk and 4½ water. B: 2½ milk and 3½ water.)**

Can A, the one that goes to London, holds 1½ pints of milk and 4½ of water. That is **1 part milk to 3 parts water**.

Try other amounts and it comes out the same, as long as there is more water than milk to start with, but not more than three times as much. Outside that, one of the pours cannot be done.

NO. 3 · MEDIUM

THE TRUSSES OF HAY

Farmer Tompkins had five trusses of hay, which he told his man Hodge to weigh before delivering them to a customer. The stupid fellow weighed them two at a time in all possible ways, and informed his master that the weights in pounds were 110, 112, 113, 114, 115, 116, 117, 118, 120, and 121. Now, how was Farmer Tompkins to find out from these figures how much every one of the five trusses weighed singly? The reader may at first think that he ought to be told "which pair is which pair," or something of that sort, but it is quite unnecessary. Can you give the five correct weights?

In plain English: Five bundles of hay are weighed two at a time, in every possible pair. The ten weights, in pounds, are 110, 112, 113, 114, 115, 116, 117, 118, 120 and 121. What does each bundle weigh?

54, 56, 58, 59 and 62 lbs.

Call the bundles A, B, C, D and E, from lightest to heaviest.

1. **Every bundle is in 4 of the 10 pairs**, once with each of the others. So adding up all ten weights counts every bundle 4 times. The ten add up to 1,156, and 1,156 ÷ 4 = **289 lbs for all five**.
2. The lightest pair must be the two lightest bundles: **A + B = 110**.

3. The heaviest pair must be the two heaviest: **D + E = 121**.

4. That accounts for four of the five bundles: 110 + 121 = 231. So **C = 289 − 231 = 58**.

5. The second-lightest pair must be A with C, since swapping in anything else makes it heavier. So A + C = 112, and **A = 112 − 58 = 54**.

6. Then **B = 110 − 54 = 56**.

7. The second-heaviest pair must be C with E, for the same reason. So C + E = 120, and **E = 120 − 58 = 62**.

8. And **D = 121 − 62 = 59**.

Check it. 54 + 56 + 58 + 59 + 62 = 289, and weighing them in pairs gives exactly the ten weights on Hodge's list.

NO. 4 · MEDIUM

A PRINTER'S ERROR

In a certain article a printer had to set up the figures $5^4 \times 2^3$*, which, of course, means that the fourth power of 5 (625) is to be multiplied by the cube of 2 (8), the product of which is 5,000. But he printed* $5^4 \times 2^3$ *as 5423, which is not correct. Can you place four digits in the manner shown, so that it will be equally correct if the printer sets it up aright or makes the same blunder?*

In plain English: Find four digits where the first to the power of the second, times the third to the power of the fourth, makes the four-figure number you get by writing the digits in a row.

$2^5 \times 9^2$ = 2592.

There is no shortcut formula for this one. It is a hunt, but there is a way to keep the hunt short: use the first two digits to pin down the answer, then see what the last two must be.

1. Try 2 and 5 as the first two digits. $2^5 = 2 \times 2 \times 2 \times 2 \times 2 =$ **32**.

2. The four-figure number starts 25, so it is somewhere from **2,500 to 2,599**.

3. So 32 times the second part has to land in that range. 2,500 ÷ 32 is just over 78, and 2,599 ÷ 32 is just over 81. The second part must be between those.

4. The second part is a digit raised to a power. The only one between 78 and 81 is **81**, which is 9^2 or 3^4.

5. 32 × 81 = 2,560 + 32 = **2,592**. With 9^2, the four digits in a row are 2, 5, 9, 2: **2592**. It works. With 3^4 they would read 2534, which does not.

Run the same test with every other pair of first digits and nothing else fits. Dudeney said this was the only answer, and we checked every possibility to be sure.

NO. 5 · HARD

THE CLOTHES LINE PUZZLE

A boy tied a clothes line from the top of each of two poles to the base of the other. He then proposed to his father the following question. As one pole was exactly seven feet above the ground and the other exactly five feet, what was the height from the ground where the two cords crossed one another?

2 feet 11 inches, however far apart the poles are.

The puzzle does not say how far apart the poles are, so choose a distance. Say 12 feet. Now walk from the tall pole towards the short one and watch both lines.

1. The line from the top of the tall pole comes down 7 feet over the 12 feet, so it **drops 7 inches for every foot** you walk.
2. The line from the bottom of the tall pole rises to 5 feet over the 12 feet, so it **climbs 5 inches for every foot**.
3. At the tall pole they start 7 feet apart, which is 84 inches. Every foot you walk closes that gap by 7 + 5 = **12 inches**.
4. 84 ÷ 12 = 7, so the lines cross **7 feet** from the tall pole.
5. By then the climbing line has gone up 7 × 5 = 35 inches, which is **2 feet 11 inches**.

Check with the other line. It started at 84 inches and dropped 7 × 7 = 49, which leaves 35. The same point.

Now move the poles 24 feet apart. The lines drop 3½ inches and climb 2½ inches a foot, closing the gap by 6 inches a foot. 84 ÷ 6 = 14 feet, and 14 × 2½ = 35 inches. The same height again. Moving the poles changes where the lines cross, but never how high.

So there is a shortcut: multiply the two heights, then divide by the two heights added together. 7 × 5 = 35, and 7 + 5 = 12. 35 ÷ 12 is 2 feet and 11 twelfths of a foot, which is 2 feet 11 inches.

NO. 6 · HARD

THE THREE GROUPS

There appeared in "Nouvelles Annales de Mathématiques" the following puzzle as a modification of one of my "Canterbury Puzzles." Arrange the nine digits in three groups of two, three, and four digits, so that the first two numbers when multiplied together make the third. Thus, 12 × 483 = 5,796. I now also propose to include the cases where there are one, four, and four digits, such as 4 × 1,738 = 6,952. Can you find all the possible solutions in both cases?

In plain English: Use each of the digits 1 to 9 exactly once to make a multiplication. Either a two-figure number times a three-figure number makes a four-figure number, or a one-figure number times a four-figure number does. There are nine in all, including the two examples. How many can you find?

There are nine, and no more.

- 12 × 483 = 5,796
- 42 × 138 = 5,796
- 18 × 297 = 5,346
- 27 × 198 = 5,346
- 39 × 186 = 7,254
- 48 × 159 = 7,632
- 28 × 157 = 4,396
- 4 × 1,738 = 6,952
- 4 × 1,963 = 7,852

> To check one, break the multiplication into easy pieces. For 39 × 186: 40 × 186 = 7,440, then take away one 186 to get **7,254**. Now tick off the digits: 3, 9, 1, 8, 6, 7, 2, 5, 4. All nine, once each.

> A tip for the hunt. The last digit of the answer comes from the last digits of the two numbers being multiplied, so some pairs rule themselves out straight away. A 1 at the end of either number, a 5 with any odd digit, or a 6 with any even digit always repeats a digit at the end of the answer. 3 × 7 ends in 1, which is fine, but 5 × 3 ends in 5, which is not.

All six are from Amusements in Mathematics (1917), which is in the public domain. The worked answers are our own.